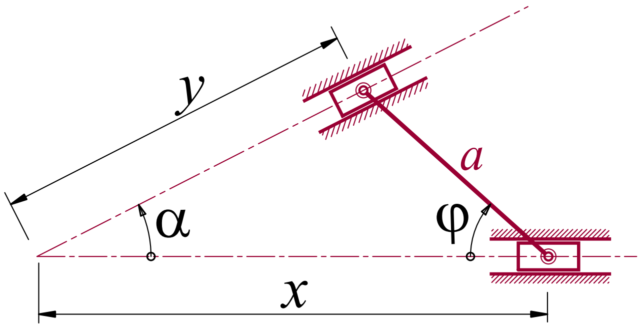
Mecanismos - 1ª Avaliação

Aluno: ______________________________

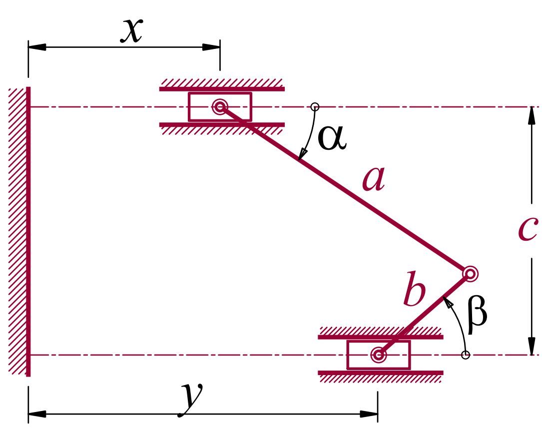
Nota:

1. Na cadeia cinemática mostrada abaixo o pistão, que se desloca horizontalmente, recebe a entrada do movimento. Determine a aceleração do pistão inclinado, designado pela coordenada generalizada y, quando a barra que tem comprimento $\boldsymbol{a}$ for perpendicular a este.



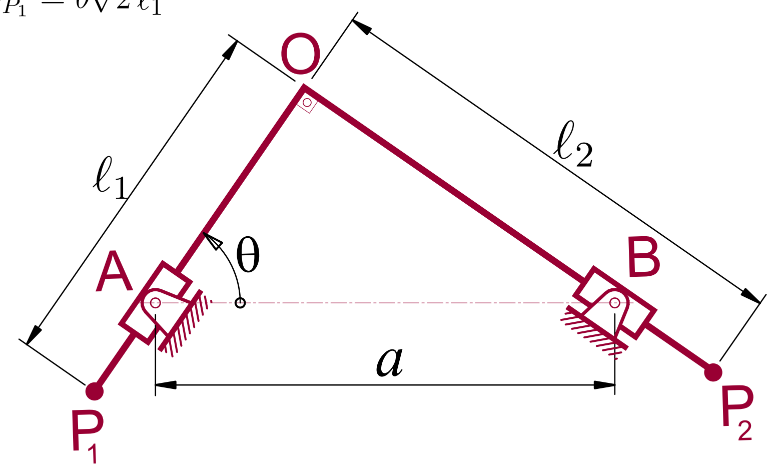
2. A cadeia mostrada tem dois graus de liberdade e os dois pistões deslizam em paralelo, o sistema tem (α, x, β, y), para coordenadas generalizadas. Verifique se os coeficientes da aceleração na coordenada α, quando a barra $\boldsymbol{a}$ se tornar perpendicular à barra $\boldsymbol{b}$, podem ser dados por:

$$\ell_{\alpha\beta} = \frac{a}{b}\left(1 - \frac{\dot{\beta}}{\dot{\alpha}}\mathrm{tg}^2\beta\right)$$
$$\ell_{\alpha y} = a\frac{\dot{\beta}}{\dot{\alpha}}\frac{\mathrm{tg}\,\beta}{\cos\beta}$$



3. Na figura, a barra rígida que forma pares prismáticos com os dois pistões em **A** e **B**, dobra a 90° no ponto **O**, seus lados têm comprimento ℓ_1 e ℓ_2. Neste problema a coordenada principal é θ. Determine as velocidades relativas entre os pontos **P**$_1$ e **P**$_2$ na horizontal e na vertical e mostre que se considerarmos $\ell_1 = \ell_2$, quando θ for igual a 45°, estas velocidades relativas serão:

$$\begin{aligned} \dot{x}_{P_2} - \dot{x}_{P_1} &= 0 \\ \dot{y}_{P_2} - \dot{y}_{P_1} &= \dot{\theta}\sqrt{2}\,\ell_1 \end{aligned}$$



Obs.:

Duas questões terão peso 3 e uma terá peso 4. Qual questão você deseja que tenha peso 4? ______

Caso o aluno não responda, a divisão será peso 3 às questões 1 e 2 e peso 4 à questão 3.

Cadeias Impostas

Deslocamento

$$\begin{cases} f_1(q, s_1, s_2, s_3, \dots, s_n) = 0 \\ f_2(q, s_1, s_2, s_3, \dots, s_n) = 0 \\ \quad \vdots \qquad\qquad\qquad \vdots \\ f_n(q, s_1, s_2, s_3, \dots, s_n) = 0 \end{cases}$$

Velocidade

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \frac{\partial f_1}{\partial s_2} & \frac{\partial f_1}{\partial s_3} & \dots & \frac{\partial f_1}{\partial s_n} \\ \frac{\partial f_2}{\partial s_1} & \frac{\partial f_2}{\partial s_2} & \frac{\partial f_2}{\partial s_3} & \dots & \frac{\partial f_2}{\partial s_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial s_1} & \frac{\partial f_n}{\partial s_2} & \frac{\partial f_n}{\partial s_3} & \dots & \frac{\partial f_n}{\partial s_n} \end{bmatrix} \quad \text{e} \quad \mathbf{F} = \begin{Bmatrix} \frac{\partial f_1}{\partial q} \\ \frac{\partial f_2}{\partial q} \\ \vdots \\ \frac{\partial f_n}{\partial q} \end{Bmatrix}$$

$$\dot{\mathbf{S}} = \begin{Bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \vdots \\ \dot{s}_n \end{Bmatrix} = \mathbf{K}\dot{q} \quad \text{sendo} \quad \mathbf{K} = \begin{Bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{Bmatrix} = -\mathbf{J}^{-1}\mathbf{F}$$

Aceleração

$$\mathbf{L} = \begin{Bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{Bmatrix} = \frac{d}{dq}\mathbf{K} \quad \text{e} \quad \ddot{\mathbf{S}} = \begin{Bmatrix} \ddot{s}_1 \\ \ddot{s}_2 \\ \vdots \\ \ddot{s}_n \end{Bmatrix} = \ddot{q}\mathbf{K} + \dot{q}^2\mathbf{L}$$

Cadeias Não Impostas

Obtenção dos F's e K's

$$\mathbf{F}_i = \begin{Bmatrix} \frac{\partial f_1}{\partial q_i} \\ \frac{\partial f_2}{\partial q_i} \\ \vdots \\ \frac{\partial f_n}{\partial q_i} \end{Bmatrix}, i = 1 \dots m \;\text{ e }\; \mathbf{K}_i = \begin{Bmatrix} k_{i1} \\ k_{i2} \\ \vdots \\ k_{in} \end{Bmatrix} = -\mathbf{J}^{-1}\mathbf{F}_i$$

Onde

$$k_{ij} = \frac{\partial s_i}{\partial q_j}$$

Velocidade e Aceleração

$$\dot{\mathbf{S}} = \sum_{i=1}^{m} \mathbf{K}_i \dot{q}_i \quad \text{e} \quad \ddot{\mathbf{S}} = \sum_{i=1}^{m} (\ddot{q}_i \mathbf{K}_i + \dot{q}_i^2 \mathbf{L}_i)$$

Onde

$$\mathbf{L}_i = \frac{1}{\dot{q}_i}\left(\sum_{j=1}^{m} \frac{\partial \mathbf{K}_i}{\partial q_j}\dot{q}_j + \sum_{k=1}^{n} \frac{\partial \mathbf{K}_i}{\partial s_k}\dot{s}_k\right)$$

Específico - Cadeias com Dois Graus de Liberdade e Cinco Barras

$$\mathbf{L}_1 = \frac{1}{\dot{q}_1}\left(\begin{bmatrix} \frac{\partial k_{11}}{\partial q_1} & \frac{\partial k_{11}}{\partial q_2} \\ \frac{\partial k_{12}}{\partial q_1} & \frac{\partial k_{12}}{\partial q_2} \end{bmatrix} \cdot \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} \frac{\partial k_{11}}{\partial s_1} & \frac{\partial k_{11}}{\partial s_2} \\ \frac{\partial k_{12}}{\partial s_1} & \frac{\partial k_{12}}{\partial s_2} \end{bmatrix} \cdot \begin{Bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{Bmatrix}\right)$$

$$\mathbf{L}_2 = \frac{1}{\dot{q}_2}\left(\begin{bmatrix} \frac{\partial k_{21}}{\partial q_1} & \frac{\partial k_{21}}{\partial q_2} \\ \frac{\partial k_{22}}{\partial q_1} & \frac{\partial k_{22}}{\partial q_2} \end{bmatrix} \cdot \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} \frac{\partial k_{21}}{\partial s_1} & \frac{\partial k_{21}}{\partial s_2} \\ \frac{\partial k_{22}}{\partial s_1} & \frac{\partial k_{22}}{\partial s_2} \end{bmatrix} \cdot \begin{Bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{Bmatrix}\right)$$

Ponto Acoplador

Deslocamento

$$\begin{Bmatrix} x_p \\ y_p \end{Bmatrix} = \begin{Bmatrix} x_o \\ y_o \end{Bmatrix} + \begin{bmatrix} \cos s & -\sin s \\ \sin s & \cos s \end{bmatrix} \cdot \begin{Bmatrix} u \\ v \end{Bmatrix}$$

Coeficiente de Velocidade

$$\mathbf{K}_p = \mathbf{K}_o + k_s \begin{bmatrix} -\sin s & -\cos s \\ \cos s & -\sin s \end{bmatrix} \cdot \begin{Bmatrix} u \\ v \end{Bmatrix}$$

Velocidade e Aceleração

$$\dot{\mathbf{X}} = \begin{Bmatrix} \dot{x}_p \\ \dot{y}_p \end{Bmatrix} = \dot{q}\mathbf{K}_p \quad \text{e} \quad \ddot{\mathbf{X}}_p = \begin{Bmatrix} \ddot{x}_p \\ \ddot{y}_p \end{Bmatrix} = \ddot{q}\mathbf{K}_p + \dot{q}^2\mathbf{L}_p$$

Onde

$$\mathbf{L}_p = \frac{d}{dq}\mathbf{K}_p$$

Gabarito

Questão 1

Coordenadas generalizadas

$$(x, \varphi, y)$$

Equações de restrição

$$\begin{cases} f_1 = y\cos\alpha + a\cos\varphi - x \\ f_2 = y\,\mathrm{sen}\,\alpha - a\,\mathrm{sen}\,\varphi \end{cases}$$

Velocidades

$$\mathbf{J} = \begin{bmatrix} -a\,\mathrm{sen}\,\varphi & \cos\alpha \\ -a\cos\alpha & \mathrm{sen}\,\alpha \end{bmatrix} \qquad \text{e} \qquad \mathbf{F} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$$

Então:

$$\mathbf{J}^{-1} = \frac{1}{a\cos(\alpha+\varphi)} \begin{bmatrix} \mathrm{sen}\,\alpha & -\cos\alpha \\ a\cos\varphi & -a\,\mathrm{sen}\,\varphi \end{bmatrix}$$

Sendo assim:

$$\mathbf{K} = -\mathbf{J}^{-1} \cdot \mathbf{F} = \begin{Bmatrix} \frac{\mathrm{sen}\,\alpha}{a\cos(\alpha+\varphi)} \\ \frac{\cos\varphi}{\cos(\alpha+\varphi)} \end{Bmatrix}$$

Ou seja:

$$k_\varphi = \frac{\mathrm{sen}\,\alpha}{a\cos(\alpha+\varphi)} \qquad \text{e} \qquad k_y = \frac{\cos\varphi}{\cos(\alpha+\varphi)}$$

Acelerações

Considerando que:

$$\ell_y = \frac{\partial k_y}{\partial x} + \frac{\partial k_y}{\partial \varphi}k_\varphi + \frac{\partial k_y}{\partial y}k_y$$

Vem:

$$\ell_y = \frac{\mathrm{sen}\,\alpha}{\cos^2(\alpha+\varphi)}k_\varphi$$

E, com:

$$\ddot{y} = k_y\ddot{x} + \dot{x}^2\ell_y$$

Obtemos:

$$\ddot{y} = \frac{\cos\varphi}{\cos(\alpha+\varphi)}\ddot{x} + \dot{x}^2\frac{\mathrm{sen}^2\,\alpha}{a\cos^3(\alpha+\varphi)}$$

Questionamento

Quando a barra que tem comprimento "***a***" for perpendicular ao pistão inclinado, tem-se um triângulo Retângulo cujos ângulos internos são α, φ e $\frac{\pi}{2}$, consequentemente, para esta situação:

$$\alpha + \varphi = \frac{\pi}{2}$$

Vemos então que $\cos(\alpha+\varphi)$ vai para zero e, em consequência disto, $\ddot{y}$ vai para infinito.

Questão 2

Coordenadas generalizadas

$$(\alpha, x, \beta, y)$$

Equações de restrição

$$\begin{cases} x + a\cos\alpha - b\cos\beta - y \\ a\,\mathrm{sen}\,\alpha + b\,\mathrm{sen}\,\beta - c \end{cases}$$

Coeficientes de Velocidade

$$\mathbf{J} = \begin{bmatrix} b\,\mathrm{sen}\,\beta & -1 \\ b\cos\beta & 0 \end{bmatrix}, \quad \mathbf{F}_\alpha = \begin{Bmatrix} -a\,\mathrm{sen}\,\alpha \\ a\cos\alpha \end{Bmatrix} \quad \text{e} \quad \mathbf{F}_x = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

Então:

$$\mathbf{J}^{-1} = \tfrac{1}{b\cos\beta} \begin{bmatrix} 0 & 1 \\ -b\cos\beta & b\,\mathrm{sen}\,\beta \end{bmatrix}$$

Sendo assim:

$$\mathbf{K}_\alpha = -\tfrac{1}{b\cos\beta} \begin{bmatrix} 0 & 1 \\ -b\cos\beta & b\,\mathrm{sen}\,\beta \end{bmatrix} \begin{Bmatrix} -a\,\mathrm{sen}\,\alpha \\ a\cos\alpha \end{Bmatrix} = \begin{Bmatrix} -\frac{a}{b}\frac{\cos\alpha}{\cos\beta} \\ -\frac{a\,\mathrm{sen}\,(\alpha+\beta)}{\cos\beta} \end{Bmatrix}$$

$$\mathbf{K}_x = -\frac{1}{b\cos\beta} \begin{bmatrix} 0 & 1 \\ -b\cos\beta & b\,\mathrm{sen}\,\beta \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Ou seja:

$$k_{\alpha\beta} = -\frac{a}{b}\frac{\cos\alpha}{\cos\beta} \quad \text{e} \quad k_{\alpha y} = -\frac{a\,\mathrm{sen}\,(\alpha+\beta)}{\cos\beta}$$

$$k_{x\beta} = 0 \quad \text{e} \quad k_{xy} = 1$$

Coeficientes da aceleração

$$\mathbf{L}_\alpha = \frac{1}{\dot\alpha}\left(\begin{bmatrix} \frac{a}{b}\frac{\mathrm{sen}\,\alpha}{\cos\beta} & 0 \\ -a\frac{\cos(\alpha+\beta)}{\cos\beta} & 0 \end{bmatrix} \begin{Bmatrix} \dot\alpha \\ \dot x \end{Bmatrix} + \begin{bmatrix} -\frac{a}{b}\frac{\cos\alpha\,\mathrm{sen}\,\beta}{\cos^2\beta} & 0 \\ \frac{a\cos\alpha}{\cos^2\beta} & 0 \end{bmatrix} \begin{Bmatrix} \dot\beta \\ \dot y \end{Bmatrix} \right)$$

$$\mathbf{L}_x = \frac{1}{\dot x}\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot\alpha \\ \dot x \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot\beta \\ \dot y \end{Bmatrix} \right)$$

Que nos leva a:

$$\ell_{\alpha\beta} = \frac{a}{b}\frac{\mathrm{sen}\,\alpha}{\cos\beta} - \frac{a}{b}\frac{\dot\beta}{\dot\alpha}\frac{\cos\alpha\,\mathrm{sen}\,\beta}{\cos^2\beta}$$

$$\ell_{\alpha y} = \frac{\dot\beta}{\dot\alpha}\frac{a\cos\alpha}{\cos^2\beta} - a\frac{\cos(\alpha+\beta)}{\cos\beta}$$

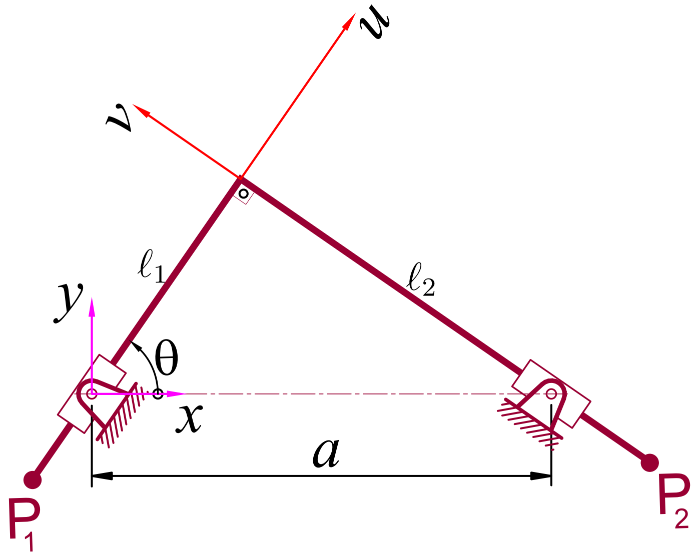
Agora vemos que quando $a \perp b$, se tem $\alpha + \beta = \dfrac{\pi}{2}$ e consequentemente $\mathrm{sen}\,\alpha = \cos\beta$, $\cos\alpha = \mathrm{sen}\,\beta$ e $\cos(\alpha+\beta) = 0$.

Logo:

$$\ell_{\alpha\beta} = \frac{a}{b}\left(1 - \frac{\dot\beta}{\dot\alpha}\mathrm{tg}^2\beta\right)$$

$$\ell_{\alpha y} = a\frac{\dot\beta}{\dot\alpha}\frac{\mathrm{tg}\,\beta}{\cos\beta}$$

Questão 3



Deslocamentos

$$\mathbf{P}_1 = \begin{Bmatrix} a\cos^2\theta \\ a\cos\theta\,\mathrm{sen}\,\theta \end{Bmatrix} + \begin{bmatrix} \cos\theta & -\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} -\ell_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \cos\theta(a\cos\theta-\ell_1) \\ \mathrm{sen}\,\theta(a\cos\theta-\ell_1) \end{Bmatrix}$$

$$\mathbf{P}_2 = \begin{Bmatrix} a\cos^2\theta \\ a\cos\theta\,\mathrm{sen}\,\theta \end{Bmatrix} + \begin{bmatrix} \cos\theta & -\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} 0 \\ -\ell_2 \end{Bmatrix} = \begin{Bmatrix} a\cos^2\theta+\ell_2\,\mathrm{sen}\,\theta \\ \frac{a}{2}\,\mathrm{sen}\,2\theta-\ell_2\cos\theta \end{Bmatrix}$$

Velocidades

Derivando-se $\mathbf{P}_1$ e $\mathbf{P}_2$ no tempo:

$$\dot{\mathbf{P}}_1 = \dot\theta \begin{Bmatrix} -a\,\mathrm{sen}\,2\theta+\ell_1\,\mathrm{sen}\,\theta \\ a\cos 2\theta-\ell_1\cos\theta \end{Bmatrix}$$

$$\dot{\mathbf{P}}_2 = \dot\theta \begin{Bmatrix} -a\,\mathrm{sen}\,2\theta+\ell_2\cos\theta \\ a\cos 2\theta+\ell_2\,\mathrm{sen}\,\theta \end{Bmatrix}$$

Vamos obter a velocidade relativa entre $\mathbf{P}_1$ e $\mathbf{P}_2$:

$$\dot{\mathbf{P}}_2-\dot{\mathbf{P}}_1 = \dot\theta\left(\begin{Bmatrix} -a\,\mathrm{sen}\,2\theta+\ell_2\cos\theta \\ a\cos 2\theta+\ell_2\,\mathrm{sen}\,\theta \end{Bmatrix} - \begin{Bmatrix} -a\,\mathrm{sen}\,2\theta+\ell_1\,\mathrm{sen}\,\theta \\ a\cos 2\theta+\ell_1\cos\theta \end{Bmatrix}\right)$$

$$\dot{\mathbf{P}}_2-\dot{\mathbf{P}}_1 = \dot\theta \begin{Bmatrix} \ell_2\cos\theta-\ell_1\,\mathrm{sen}\,\theta \\ \ell_2\,\mathrm{sen}\,\theta+\ell_1\cos\theta \end{Bmatrix}$$

Agora, quando θ = 45º, vamos ter $\mathrm{sen}\,\theta=\cos\theta=\frac{\sqrt{2}}{2}$ e ficamos com:

$$\dot{\mathbf{P}}_2-\dot{\mathbf{P}}_1 = \dot\theta\frac{\sqrt{2}}{2} \begin{Bmatrix} \ell_2-\ell_1 \\ \ell_2+\ell_1 \end{Bmatrix}$$

Agora, considerando $\ell_1 = \ell_2$, ficamos com:

$$\dot{\mathbf{P}}_2-\dot{\mathbf{P}}_1 = \dot\theta\frac{\sqrt{2}}{2} \begin{Bmatrix} 0 \\ 2\ell_1 \end{Bmatrix}$$

E, finamente, ficamos com:

$$\dot{\mathbf{P}}_2-\dot{\mathbf{P}}_1 = \begin{Bmatrix} 0 \\ \dot\theta\sqrt{2}\,\ell_1 \end{Bmatrix}$$